Left Distributive Left Idempotent Groupoids

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Definition

left distributivity	$x \cdot (y \cdot z) = (x \cdot y) \cdot (x \cdot z)$
idempotency	$x = x \cdot x$

Example

Let $(G, \cdot)$ be a group. We define on G an LDI operation:

$$x * y = xyx^{-1}.$$

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Word problem for an algebra A:
To decide whether two terms represent the same element in A

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- *The word problem of free LDI groupoids is still open*
- *The word problem of free LD groupoids solved by Patrick Dehornoy*

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Solution of the Word Problem of LD

Steps of Dehornoy's solution of the word problem of free LD grupoids

1. Construction of the geometry monoid and the geometry group
2. Description of their presentations
3. Solution of the word problem of the geometry group
4. Solution of the word problem for the free monogenerated LD groupoid
5. Solution of the word problem for the free LD groupoid with more generators

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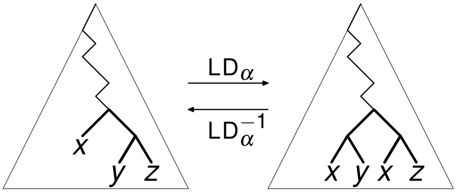
Geometric Operators



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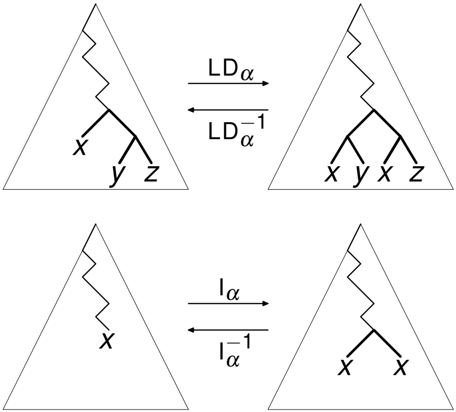

Definitions of Geometry Monoids and Geometry Groups

Definitions

The *geometry monoid* of LD is the monoid generated by all LD_α and LD_α^{-1}.
The *geometry monoid* of LDI is the monoid generated by all LD_α, I_α, LD_α^{-1} and I_α^{-1}.

Definitions

The *geometry group* of LD is the geometry monoid of LD quotioned by $\mathrm{LD}_\alpha \circ \mathrm{LD}_\alpha^{-1} = \mathrm{LD}_\alpha^{-1} \circ \mathrm{LD}_\alpha = \mathrm{id}$.
The *geometry group* of LDI is the geometry monoid of LDI quotioned by $\mathrm{LD}_\alpha \circ \mathrm{LD}_\alpha^{-1} = \mathrm{LD}_\alpha^{-1} \circ \mathrm{LD}_\alpha = \mathrm{I}_\alpha \circ \mathrm{I}_\alpha^{-1} = \mathrm{I}_\alpha^{-1} \circ \mathrm{I}_\alpha = \mathrm{id}$.

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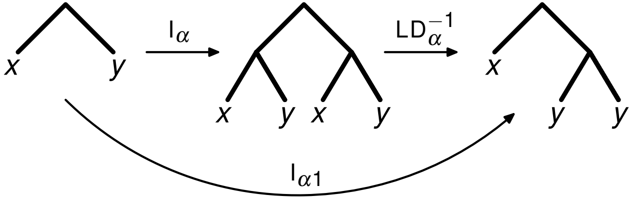
Example of a Relation in the Geometry Monoid



$$\mathsf{I}_\alpha \bullet \mathsf{LD}_\alpha^{-1} = \mathsf{I}_{\alpha 1}$$

Unconvenient Relations in the LDI Geometry Monoid

Different possibilities how to encode the expansion $t \to (t \cdot t) \cdot (t \cdot t)$ at an address α:

$$\mathrm{I}_\alpha \bullet \mathrm{I}_\alpha = \mathrm{I}_\alpha \bullet \mathrm{I}_{\alpha 0} \bullet \mathrm{I}_{\alpha 1} = \mathrm{I}_\alpha \bullet \mathrm{I}_{\alpha 1} \bullet \mathrm{I}_{\alpha 0} = \mathrm{I}_\alpha \bullet \mathrm{I}_{\alpha 1} \bullet \mathrm{LD}_\alpha$$

In the group we have

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- Right constant groupoids (satisfying $x \cdot z = y \cdot z$)
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Relation Between LDI and LDLI

Definition

The *right height* of a term is the length of its rightmost branch.

Theorem

Two terms are LDLI-equivalent if and only if they are LDI-equivalent and they have the same right height.

Open Question

$\mathcal{V}_{LDLI} = \mathcal{V}_{LDI} \vee \mathcal{V}_{RC}$?

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Presentation of the Syntactical Monoid of LDLI

Operators in the geometry monoid of LDLI satisfy

$$\begin{aligned}
\mathrm{LD}_{\gamma 0\alpha} \cdot \mathrm{LD}_{\gamma 1\beta} &= \mathrm{LD}_{\gamma 1\beta} \cdot \mathrm{LD}_{\gamma 0\alpha} \\
\mathrm{I}_{\gamma 0\alpha} \cdot \mathrm{LD}_{\gamma 1\beta} &= \mathrm{LD}_{\gamma 1\beta} \cdot \mathrm{I}_{\gamma 0\alpha} \\
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\end{aligned}$$

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Complemented Presentation

Definition

A presentation of a monoid is called *complemented* if
(i) for each pair of different letters x, y there exist at most one relation $xu = yv$;
(ii) for each letter x there exist no relation $xu = xv$.

Fact

The syntactical monoid of LDLI has a complemented presentation.

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Monoids with complemented presentations can be studied using the word revesing *method.*

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Properties of the Syntactical Monoid of LDLI

The syntactical monoid of LDLI has following properties:

- Every two elements have the least common right multiple
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- The monoid is left cancellative
- The monoid has a decidable word problem
- The monoid does not embed into the geometry monoid of LDLI

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Further Questions

Open Question

Is the syntactical monoid right cancellative?
Does the syntactical monoid embed into its group of fractions?

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Is there some mapping from the terms to the syntactic group that preserves (somehow) the LDLI equivalence?

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Does there exist an integer n such that the word problem for all free LDLI groupoids can be intepreted in the free n-generated LDLI groupoid?

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History of LDLI

left (self)distributivity	$x \cdot (y \cdot z) = (x \cdot y) \cdot (x \cdot z)$
left (pseudo)idempotency	$x \cdot y = (x \cdot x) \cdot y$

This combination was first (and only) studied by T. Kepka and P. Němec (1998 resp. 2001). They used it to study finite simple LD groupoids and left distributive left quasigroups.

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Left Cancellative LDLI Groupoids

Theorem (T. Kepka, P. Němec; P. Dehornoy)

Every left cancellative left idempotent left distributive groupoid embeds into a left distributive left quasigroup of the same cardinality.

Corollary

$\mathcal{V}_{LCLDLI} = \mathcal{V}_{LDLQ}$.

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Equality of Varieties

Theorem (D. Larue; D. Stanovský)

The following varieties coincide:

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- *LDI left quasigroups*
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Idempotent congruence

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For a groupoid G, the smallest equivalence satisfying $(a, a \cdot a) \in ip_G$, for all $a \in G$, is denoted ip_G.

Proposition

For an LDLI groupoid G, the equivalence ip_G is a congruence, the factor G/ip_G is an LDI groupoid and each congruence class is a connected right constant groupoid.

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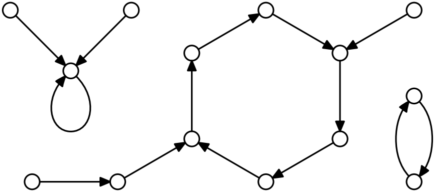

A disconnected right constant groupoid

Construction of LDLI Groupoids

Proposition

There exists a construction of each LDLI groupoid from an LDI groupoid and a set of connected right constant groupoids.

Open Question

Is every congruence class of ip_G in the free LCLD groupoid a left cancellative RC groupoid?

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The End of the Presentation

Thank you for your attention